On the Relationship of the Mechanical Properties of Soils and Rocks to the Velocity of Elastic Waves

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Abstract

The velocities of the elastic longitudinal and transverse waves in soils and rocks are measured using the ultrasonic pulse method. The comparison of the dynamic elastic constants of soils and rocks determined by the ultrasonic method with static ones obtained by the uniaxial compression test, the possibility of estimating their compressive strength from the dynamic shear modulus and the effect of pore fluids on the ultrasonic volocity in soils are investigated. From these results, the following conclusions are drawn: (1) for relatively isotropic material such as soil and mortar, the dynamic shear modulus G_d increases with the increasing of the compressive strength q_u and q_u/G_d becomes larger with q_u, (2) the velocity of longitudinal wave V_l in soils is affected by pore fluids and the presence of air in the pore causes the decrease in V_l, (3) Young's dynamic modulus E_d increases with the increasing of the static one E_s and it seems that the relationship between E_d and E_d/E_s for a soil is approximately represented by hyperbola.

1. Introduction

Recently the survey of the ground by the velocity of the elastic waves from which the dynamic elastic constants of soil and rock constituting the ground are calculated has come to be widely applied. We are to try to make an estimate of the static properties of the ground from its dynamic properties. Though the dynamic elastic constants of soils and rocks calculated from the velocity of the elastic waves do not agree with the static ones because they are not perfect elastic bodies, there may be a mutual relation between these elastic constants. Therefore it may indicate the possibility of estimating the static elastic constants and the strength of soils or rocks from the velocity of elastic waves. As soils and rocks are very complicated in their constitution, the velocity of elastic longitudinal and transverse waves in them are affected by many factors, for example, the structure of the soil skeleton, properties of pore fluids, properties of soil solid, effective stress, etc. in the case of a soil.

Many experimental investigations have been made of the factors which affect the velocity of the elastic waves in soils and rocks. Wyllie, Gregory and Gardner[1] investigated the effect of pressure, pore fluids and porosity on the velocity of the longitudinal wave and Gregory[2] investigated the effect on the velocity of the transverse wave in the porous rock. Their results show that the velocity increases with the increasing of pressure applied to the specimens and with the decreasing of porosity, and that the effect of oil and gas saturation are comparatively minor. Ide[3] found that the values of Young's dynamic modulus for granite samples are higher than those determined statically. Ide's results were later confirmed by several experiments using different rocks.

The purpose of this paper is to investigate the effect of pore fluids on the velocity of the elastic wave mainly in clay, the possibility of estimating the strength of soils and rocks from the dynamic elastic constants and the comparison of the dynamic elastic constants with static ones, using the ultrasonic pulse method.

2. Apparatus and Method of Experiments

The velocity of the elastic waves in soils and rocks are measured using the ultrasonic pulse method. The frequencies of the compressive mode and shearing mode barium titanate transducer are 50 kc/sec and 30 kc/sec respectively. A high capacity triaxial testing machine was used for the consolidation of the clay sample under allround pressure and a uniaxial testing machine with a capacity of 30 tons for deformation and strength tests on rocks, mortars and soils.

In the experiment, cylindrical specimens about 3.6 cm in diameter by 8.0 cm long for the clays and the mortars, and about 5.0 cm in diameter by 10.0 cm long for rocks were used. Each specimen was greased on its end platten when it was compressed, to prevent the influence of end friction on the static elastic constants and the strength of the materials. The uniaxial compressive tests were carried out with a rate of strain of 1%/min for clays and of 0.2%/min for mortars and rocks. For such rates of strain the difference in measured compressive strength or Young's modulus may be negligible.

3. Sample

3-1 Clay Sample

The following clay samples were used in these experiments; (1) saturated

Table 1. Physical properties of clays.

	Sample-A	Sample-B
Specific gravity	2.61	2.69
Liquid limit	64.5%	4.33%
Plastic limit	28.2%	21.3%

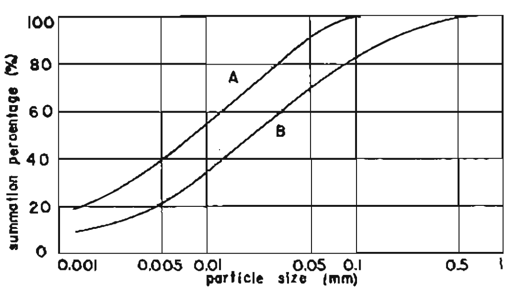

Fig. 1. Particle size distribution curve.

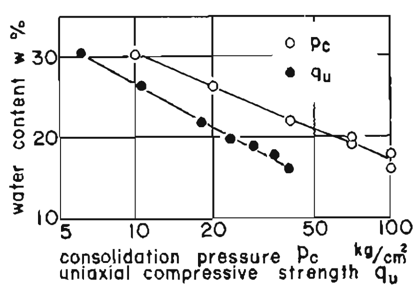

Fig. 2. Relationship between water content and consolidation pressure or uniaxial compressive strength.

clays which were consolidated under various pressures in the laboratory, (2) unsaturated clays which were dried in air, (3) unsaturated clays which were compacted with different water contents. The samples in (1) and (2) are the same clay (called sample-A) and in (3) are the other (called sample-B). The physical properties of these samples are shown in Table-1 and Fig. 1. The relationships between water content and consolidation pressure or uniaxial compressive strength for sample-A are shown in Fig. 2.

3-2 Mortar

Instead of the rock sample, mortar was used, because it was difficult to obtain a uniform rock sample. In order to obtain mortars with different strengths, the mixing ratio of sand to cement in mortar was changed within the limit from 1 : 0.5 to 1 : 4.

3-3 Rock Sample

As rock samples, shale with clearly parallel sedimentation cut different direction to sedimentation and basalt was used.

4. Test Results and Consideration

Assuming the materials as elastic bodies, we can calculate the dynamic elastic constants from the velocity of the elastic waves in the materials using the following equations.

$$V_l=\sqrt{\frac{\lambda+2\mu}{\rho}}, \qquad V_s=\sqrt{\frac{\mu}{\rho}} \qquad \cdots\cdots\cdots(1)$$

$$E_d=\frac{\mu(3\lambda+2\mu)}{\lambda+\mu} \qquad \cdots\cdots\cdots(2)$$

$$\mu_d=\frac{\lambda}{2(\lambda+\mu)} \qquad \cdots\cdots\cdots(3)$$

$$K_d=\lambda+\frac{\mu}{3} \qquad \cdots\cdots\cdots(4)$$

$$G_d=\mu=\frac{E_d}{2(1+\mu_d)} \qquad \cdots\cdots\cdots(5)$$

where

V_l, V_s: propagation velocity of longitudinal and transverse waves respectively.

λ, μ : Lame constant

E_d : Young's dynamic modulus

μ_d : Poisson's dynamic ratio

K_d : dynamic bulk modulus

G_d : dynamic shear modulus

4-1 Effect of Dry Density on the Velocity

The strength and Young's modulus etc. of soil depend on its dry density γ_d. Then the results of the experiment on the effect of dry density on the velosity of ultrasonic longitudinal V_l and transverse V_s waves for the saturated clay are shown in Fig. 3. From Fig. 3 the velocity of longitudinal wave V_l has a

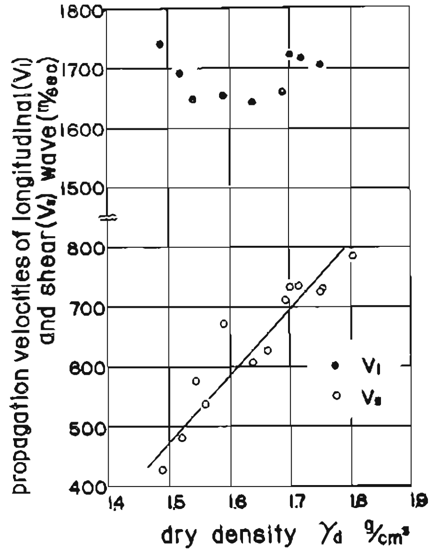

Fig. 3. Relationship between velocity of longitudinal or transverse wave and dry density.

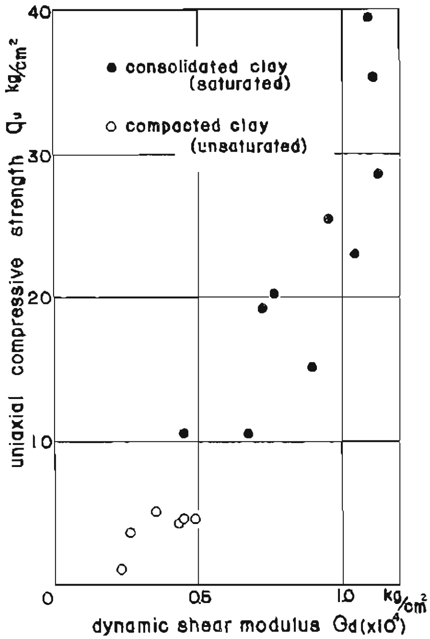

Fig. 4. Relationship between uniaxial compressive strength and dynamic shear modulus for clays.

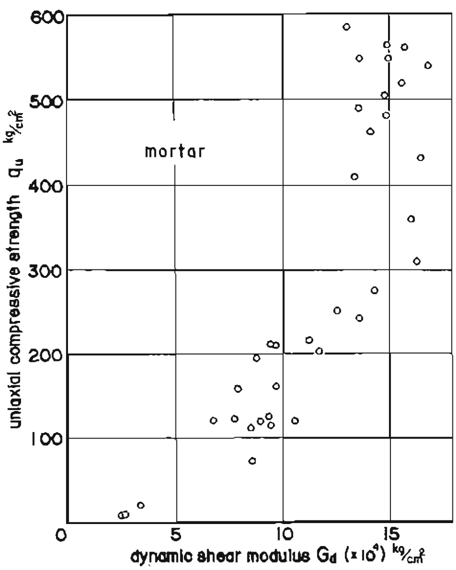

Fig. 5. Relationship between uniaxial compressive strength and dynamic shear modulus for mortar.

minimum value at some dry density. This fact shows that V_l is affected not only by the compressibility of the soil skeleton but also by that of pore fluids.

Fig. 3 also shows that the velocity of transverse wave V_s linearly increases with the increasing of the dry density. It seems that V_s depends on the dry density or the strength of the soil skeleton and is almost un affected by pore fluids. The details about the effect of pore fluids on the velocity of ultrasonic waves in soil will be described later.

4-2 Estimation of Compressive Strength of Soils and Rocks from Velocity of Ultrasonic Waves

As mentioned in 3-1, the velocity of the transverse wave is shown as a linear function of dry density for the saturated soil which has a close relation to its strength. Therefore, we have the possibility of estimating the strength of soils and rocks from V_s. Then, the relationship between uniaxial compressive strength q_u and dynamic shear modulus G_d which can be calculated from V_s and unit weight ρ only from Eq. (5) is as shown in Fig. 4 for the clays. Fig. 4 contains the test result for compacted unsaturated clay. Though the experimental values are some what scattered, q_u increases with the increasing of G_d and the ratio q_u/G_d also increases with G_d. In order to investigate this fact for the material which has larger strength, a mortar which is comparatively isotropic like a soil was used in our test. The test result for mortar is shown in Fig. 5. The relationship between q_u and G_d for the mortar in Fig. 5 is similar to that for clay in Fig. 4. It is seen that the dynamic shear modulus G_d hardly changes with the increasing of uniaxial compressive strength q_u at the value of G_d of 1.1×10^4 kg/cm^2 for clay and of 15×10^4 kg/cm^2 for mortar. We have not found out the cause of this fact, but there may be some errors in the measurement of the velocity of the ultrasonic waves. It is clear that the relationship between q_u and G_d, which was represented by a straight line in the previous report (4), is not linear, although it is difficult to obtain an empirical equation representing the relationship between them.

Generally rocks have beds and are then unisotropic. Therefore, the measurement of the velocity of the ultrasonic waves and the uniaxial compressive strength for shale which has very thin beds, were carried out along the three different directions; perpendicular to the bed, parallel to the bed and at an angle of 45° to the bed. The basalt has a weakened bed of random direction. The test results for these rocks are given in Fig. 6 showing the relationship between uniaxial compressive strength q_u and dynamic shear modulus G_d. The experimental values are very scattered because of the anisotropy and unhomogeneity of the rock samples. Therefore, we can not estimate the uniaxial compressive strength q_u from the dynamic shear modulus G_d.

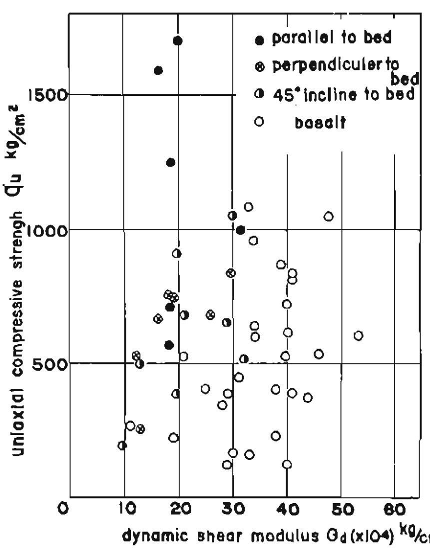

Fig. 6. Relationship between uniaxial compressive strength and dynamic shear modulus for rocks.

4-3 *Effect of Pore Fluids on the Velocity of Ultrasonic Waves*

The velocity of elastic waves in a soil depends on the degree of saturation. Then we measured the velocity of ultrasonic longitudinal and transverse waves in the unsaturated clays during the drying process in air. These clays were

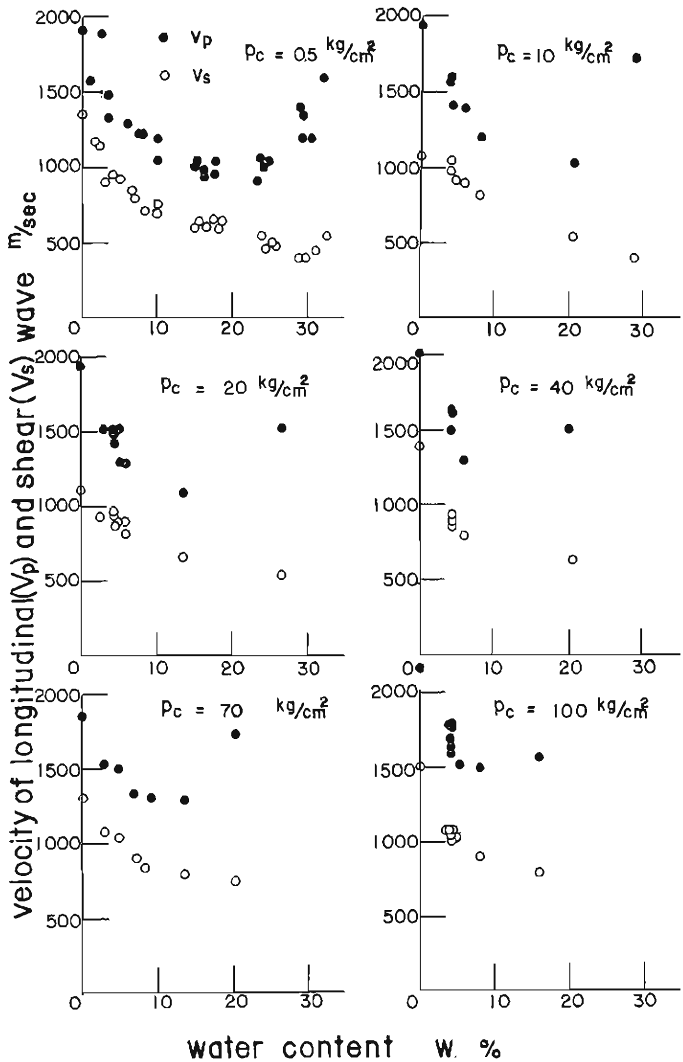

Fig. 7. Relationship between velocity of longitudinal or transverse wave and water content for unsaturated clay.

consolidated under the different pressures $p_c=0.5$, 10, 20, 40, 70, 100 kg/cm² before they were dried in order to obtain clay samples which have the same water content at different degrees of saturation. In Fig. 7 the variations in the velocities of longitudinal V_l and transverse V_s waves with water content w are shown. Fig. 7 shows that the variation in V_l with w is similar to that in Fig. 3 for saturated clay. The velocity of longitudinal wave V_l in a soil may be influenced by the compressibility of pore fluids, soil skeleton and soil solid. The smaller their compressibility is, the higher the V_l. In the unsaturated soil during the drying process, the compressibility of the soil solid may be unchanged and that of pore fluids (water and air are mixed) increases with the decreasing of that of the soil skeleton. Therefore, V_l has a minimum value at some water content from 10% to 20% as shown in Fg. 7.

The result in Fig. 7 in the case of a water content larger than 15% agrees qualitatively with Whitman's result[5] which shows that the velocity of the longitudinal wave increases with the increasing of the water content for the compacted unsaturated clay within the limit of a water content from 15% to 35%.

On the other hand, Fig. 7 shows that the velocity of the transverse (shear) wave V_s increases with the decreasing of the water content. This is due to the fact that pore fluids have no shearing resistance and that the shearing resistance of the soil skeleton increases with the decreasing of the water content.

The velocity of the longitudinal and the transverse waves in the soil is affected by the degree of saturation in addition to the dry density and water content. We will investigate the influence of the degree of saturation on the velocities of longitudinal and transverse waves in the soil. We can obtain the relationship between the water content and the degree of saturation during the drying process of the clay in Fig. 8. The relationships between V_l, V_s obtained from Fig. 7 and S_r from Fig. 8 at constant water contents of 15% and 20% with interpolation are shown in Fig. 9. It is clear from Fig. 9 that V_l is obviously affected by the degree of saturation, that is, by the compressibility

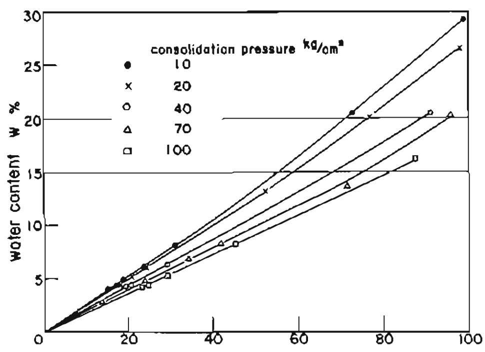

Fig. 8. Relationship between water content and degree of saturation for unsaturated clay during drying process.

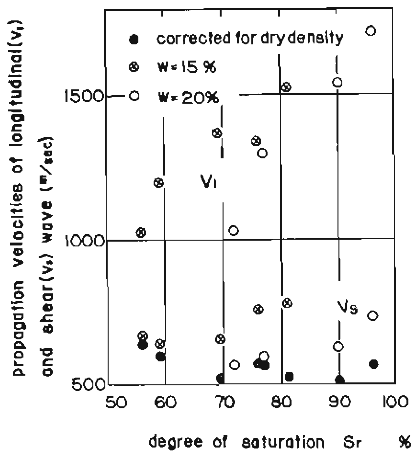

Fig. 9. Relationship between velocity of longitudinal or transverse wave and degree of saturation for unsaturated clay.

Table 2. Effect of pore saturant on ultrasonic wave velocity ft/sec (by Gregory)

Pore saturant	P-Wave velocity	S-Wave velocity
Air	8,500	6,690
Oil	10,930	5,810
CCl_4	10,250	5,630

of the pore fluids. V_s is also affected by the degree of saturation. But each value of V_s must be corrected for the dry density, because each clay sample has a different dry density which increases with the increasing of the degree of saturation at the same water content. Each value of V_s in Fig. 9 was corrected for the dry density, by means of assuming the relationship between velocity of transverse wave V_s and dry density for the saturated clay in Fig. 3 to be effective for the unsaturated clay, and the amount of increase of each V_s due to the amount of increase of the dry density of each plot above the minimum value of V_s in Fig. 9 obtained from Fig. 3 was subtracted from each value of V_s in Fig. 9. The corrected values of V_s are plotted against the degree of saturation marked with the point. Though this relationship is qualitative, the corrected values of V_s which are considered to be the velocity of the transverse wave in clay at the same dry density, slightly increase with the decreasing of the degree of saturation in contrast to the value of V_s before correction. This fact seems to agree with Gregory's[2] result represented in Table-2 which shows that the velocity of the transverse wave (s-wave) in the rock saturated with gas is higher than that saturated with a liquid such as oil or CCl_4 in contrast to the velocity of the longitudinal wave.

4-4 Comparison of Young's Dynamic Modulus with the Static One

As stated in the introduction, Young's dynamic modulus of materials is higher than the static one. Young's dynamic modulus was calculated from Eq. (2) and the static one was determined from the gradient of the initial straight line parts in the stress-strain curve obtained by the uniaxial compression test.

Young's dynamic modulus E_d is plotted against the static one E_s for the

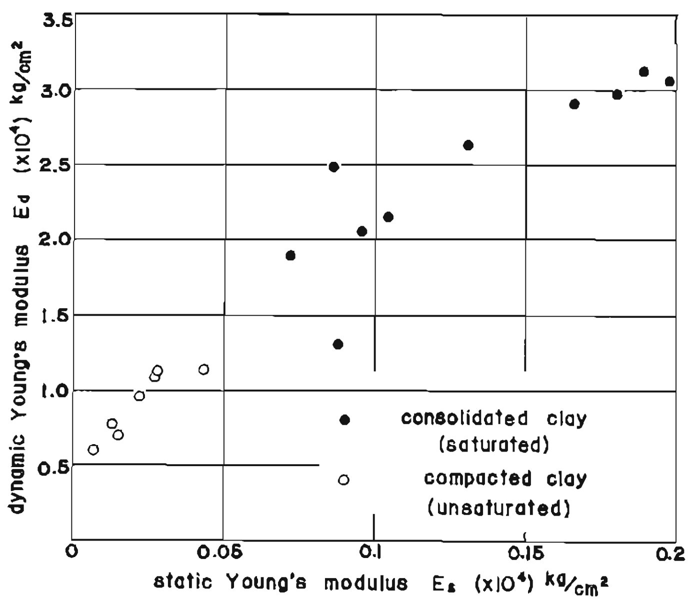

Fig. 10. Relationship between Young's dynamic modulus and the static one for clays.

consolidated saturated (sample-A) and the compacted unsaturated (sample-B) clays in Fig. 10. There may be a mutual relation between them and it becomes evident from Fig. 10 that E_d is larger than E_s and that the ratio E_d/E_s becomes smaller as E_s becomes larger. Then the ratio E_d/E_s is plotted against Young's dynamic modulus E_d for these clays in Fig. 11. From this figure, it seems that the relationship between them is approximately represented by hyperbola with some scattering in the plots. For the consolidated clay of E_d in the limit of $2\sim3\times10^4$ kg/cm² the ratio E_d/E_s becomes about 20～30 and for the compacted clay of E_d smaller than 1×10^4 kg/cm² it shows a rapid increase to become about 40～80.

For the mortar which has a larger Young's modulus, the ratio E_d/E_s is plotted against Young's dymamic modulus E_d in Fig. 12. For a material of E_d in the limit of $2\sim4\times10^5$ kg/cm² such as mortar, the ratio E_d/E_s becomes about 2～6 and shows a rapid increase in the smaller E_d. Though it is expected that the relationship between E_d/E_s and E_d for mortar is approximately represented by hyperbola as well as soils, the conclusion cannot be drawn due to the insufficient number of suitable experimental samples.

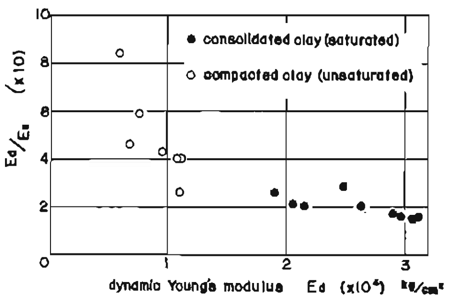

Fig. 11. Relationship between Young's modulus ratio E_d/E_s and Young's dynamic modulus for clays.

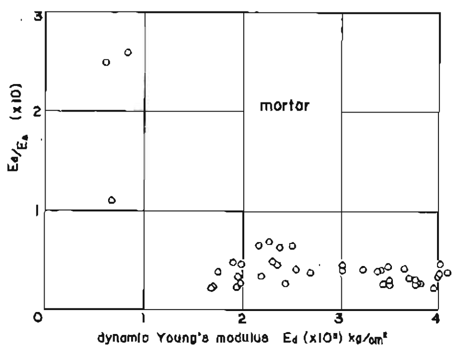

Fig. 12. Relationship between Young's modulus ratio E_d/E_s and Young's dynamic modulus for mortar.

4-5 *Poisson's Dynamic Ratio*

Poisson's static ratio for a saturated clay obtained by the uniaxial compression test at a strain rate of 1%/min may become near 0.5, because the compressibility of the soil skeleton is very high compared with those of soil solid and pore water, and pore water can not flow into or out of a specimen during the test due to the very low permiability of the clay. Poisson's dynamic ratio μ_d calculated from Eq. (3) is plotted against consolidation pressure p_c for the saturated clay in Fig. 13. Poisson's dynamic ratio μ_d becomes smaller according as the consolidation pressure becomes larger or water content lower. Though μ_d can not be compared with Poisson's static ratio because the static one was not obtained by the uniaxial compressive test, the difference between them may be considered to be small for the saturated clay. This is clear from the previous report[4] in which it is represented that most values of Poisson's dynamic ratio between 0.45 and 0.5 were obtained for the undisturbed

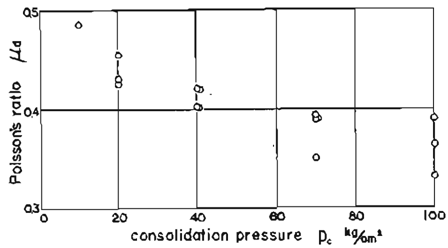

Fig. 13. Relationship between Poisson's dynamic ratio and consolidation pressure for saturated clay.

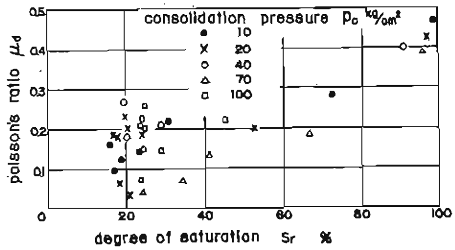

Fig. 14. Relationship between Poisson's dynamic ratio and degree of saturation.

saturated clay.

On the other hand, for the unsaturated clay during the drying process Poisson's dynamic ratio is plotted against the degree of saturation in Fig. 14, which shows that it is very sensitive to the degree of saturation, though these values are very scattered especially at low degrees of saturation.

5. Conclusion

1) For relatively isotropic material such as soil or mortar the mutual relation is observed between the uniaxial compressive strength q_u and the dynamic shearing modulus G_d and the ratio q_u/G_d becomes smaller according as G_d becomes smaller. But for unisotropic material such as sedimentary rock there is no mutual relation between them.

2) Pore fluids in a clay affect the velocity of the longitudinal wave and a little that of the transverse wave which has a linear relation with the dry density.

3) The mutual relation is observed between Young's dynamic modulus E_d and the static one E_s and the relationship between the ratio E_s/E_d and E_d is approximately represented by a hyperbola.

4) Poisson's dynamic ratio for a clay becomes smaller according as the degree of saturation becomes lower.

For a future study, we shall leave the investigation of the influence of pore pressure and effective stress in soil on the velocity of elastic waves, and the relation of the cohesion parameter and the angle of internal friction of soil to them.

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